RESEARCH ARTICLE

On Fuzzy Supra β −Baire Spaces

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ABSTRACT: In this paper, the concepts of fuzzy supra β −nowhere dense set, fuzzy supra β −first category set and fuzzy supra β −second category set in fuzzy supra topological spaces are introduced and studied. By means of fuzzy supra β −nowhere dense sets, the concept of fuzzy supra β −Baire space is defined and several characterizations of fuzzy supra β −Baire spaces are studied.

KEYWORDS: Fuzzy supra open set, fuzzy supra dense set, fuzzy supra β −open set, fuzzy supra β −nowhere dense set, fuzzy supra β −first category set, fuzzy supra β −Baire space.

https://doi.org/10.29294/IJASE.5.1.2018.846-850

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1. INTRODUCTION

In 1965, Zadeh[1] introduced the concept of fuzzy sets as a new approach for modeling uncertainties. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. Topology provided he most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by Chang [2] in 1968. The paper Chang paved the way for the subsequence tremendous growth of the numerous fuzzy topological concepts. Several mathematicians have tried all the pivotal concepts of general topology for extension to the fuzzy setting.

Abd El- Monsef at al. [3] introduced Fuzzy supra topological spaces generalizing classical supra topological spaces introduced by Mashhour et al [4].The concept of fuzzy supra σ −Baire Spaces were introduced and studied by Poongothai and Thangaraj [5]. In this paper, the concepts of fuzzy supra β −Baire spaces are introduced and studied. Several characterizations of fuzzy supra β −Baire spaces are studied. In used for the relationship between these new concepts and other types of fuzzy supra baire spaces.

2. PRELIMINARIES

Now we introduce some basic notions and results used in the sequel. The basic operations on fuzzy sets are taken as usual.

Definition 2.1 [7]: A fuzzy topology T on a set X is a family of fuzzy sets in X such that

1. $0_X, 1_X \in T$;
2. $A, B \ \in T \Rightarrow A \wedge B \in T$ and
3. $A_i \in T \Rightarrow \vee A_i \in T$

The pair (X, T) is called a fuzzy topological space (FTS). The elements of T are called fuzzy open sets and the complement of fuzzy open set is called fuzzy closed set.

Definition 2.2 [6]: The closure and the interior of a fuzzy set A in FTS (X, T) are denoted and defined respectively as

$$Cl(A) = \wedge \{B : A \le B, B \text{ is a fuzzy closed set in } X\},$$
$$Int(A) = \vee \{B : B \le A, B \text{ is a fuzzy open set in } X\}.$$

Definition 2.3[3]: A collection T^* of fuzzy open sets in a set X is called a fuzzy supra topology on X if the following conditions are satisfied:

$$(1) 0_X, 1_X \in T \quad and \quad (2) A_i \in T \Rightarrow \vee A_i \in T$$

The pair (X, T^*) is called a fuzzy supra topological space (FSTS). The elements of T^* are called fuzzy supra open sets (FSOS) and the complement of a fuzzy supra open set is called fuzzy supra closed set (FSCS). The collection of all fuzzy supra open sets (resp.fuzzy supra closed sets) of the FSTS (X, T^*) is denoted by FSOS(X) (resp. FSCS(X)).

Remark 2.4 [3]:

1. Every FTS is a FSTS.
2. If (X, T^*) is an associated FSTS with the FTS (X, T) (i.e. $T \subseteq T^*$), then every fuzzy open (closed) set in the FTS (X, T) is fuzzy supra open (closed) set in the FSTS (X, T^*).

Definition 2.5 [4]: Let (X, T^*) is a FSTS and A be fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior are denoted and defined respectively as

$$Cl^*(A)$$
$$= \wedge \{B : A \le B, B \text{ is a fuzzy supra closed set in } X\},$$
$$Int^*(A)$$
$$= \vee \{B : B \le A, B \text{ is a fuzzy supra open set in } X\}.$$

Remark 2.6 [4]:

1. The fuzzy supra closure of a fuzzy set A in a FSTS is the smallest fuzzy supra closed set containing A.
2. The fuzzy supra interior of a fuzzy set A in a FSTS is the largest fuzzy supra open set containing in A.
3. If (X, T^*) is an associated FSTS with the FTS (X, T) and A is any fuzzy set in X, then

$$Int(A) \le int^*(A) \le A \le Cl^*(A) \le Cl(A).$$

Properties of fuzzy supra closure and fuzzy supra interior which are needed in the sequel, are summarized in the following theorem.

Theorem 2.7 [8]: For any fuzzy sets A and B in a FSTS (X, T^*).

1. $A \in FSCS(X) \Leftrightarrow Cl^*(A) = A$,
2. $A \in FSOS(X) \Leftrightarrow Int^*(A) = A$;
3. $A \le B, \Rightarrow Cl^*(A) \le Cl^*(B) and\ Int^*(A) \le Int^*(B)$;
4. $Cl^*(Cl^*(A)) = \ Cl^*(A)\ and\ Int^*(Int^*(A)) = Int^*(A)$;

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Received: 05.06.2018 **Accepted:** 10.08.2018 **Published on**: 24.08.2018

(5) $Cl^*(A \vee B) \geq Cl^*(A) \vee Cl^*(B)$;
(6) $Cl^*(A \wedge B) \leq Cl^*(A) \wedge Cl^*(B)$;
(7) $Int^*(A \vee B) \geq Int^*(A) \vee Int^*(B)$;
(8) $Int^*(A \wedge B) \leq Int^*(A) \wedge Int^*(B)$;
(9) $Cl^*(A^C) = \left(Int^*(A)\right)^C, Int^*(A^C) = \left(Cl^*(A)\right)^C$.

Definition 2.8[9] :Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A of X is called fuzzy supra β −open if $A \leq cl^*int^*cl^*(A)$. The complement of a fuzzy supra β −open set is called fuzzy supra β −closed.

Definition 2.9[9]: Let (X, T^*) be a fuzzy supra topological space and A be any fuzzy set in (X, T^*). Then the fuzzy supra $\beta-$ interior and the fuzzy supra β −closure of A are defined as follows:

$\beta - cl^*(A)$
$= \wedge \{B : A \leq B, B \text{ is a fuzzy supra closed set in } X\}$,
$\beta - Int^*(A)$
$= \vee \{B : B \leq A, B \text{ is a fuzzy supra open set in } X\}$.

Definition 2.10 [5]: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra nowhere dense if there exists no non-zero fuzzy open set B in (X, T^*) such that $B < cl^*(A)$. That is, $int^*cl^*(A) = 0$.

3. FUZZY SUPRA $\beta-$ NOWHERE DENSE SETS

Definition 3.1: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A in (X, T^*) is called a fuzzy supra β −nowhere dense set if there exists no non-zero fuzzy β −open set B in (X, T^*) such that $B < \beta - cl^*(A)$ .That is, $\beta - int^*\beta - cl^*(A) = 0 \; in \; (X, T^*)$.

Example 3.2: Let $X = [a, b, c\}$. The fuzzy sets λ, μ and ν are defined on X as follows:
$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 1, \lambda(b) = 0.2, \lambda(c) = 0.7$.
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.3, \mu(b) = 0.1, \mu(c) = 0.2$.
$\vartheta : X \to [0,1]$ is defined as $\vartheta(a) = 0.7, \vartheta(b) = 0.4, \vartheta(c) = 1$.
Then $T^* = \{0, \lambda, \mu, \nu, \lambda \wedge \nu, \lambda \vee \nu, 1\}$ is a fuzzy supra topology on X. On computation, one can see that the non-zero fuzzy supra β −nowhere dense sets in (X, T^*) are $1 - \lambda, 1 - \mu, 1 - \vartheta$ .The fuzzy supra β −nowhere dense sets in (X, T^*) are $1 - \lambda, 1 - \mu, 1 - \vartheta$.

Proposition 3.3: If A is a fuzzy supra β −closed set with $\beta - int^*(A) = 0$, in a fuzzy supra topological space (X, T^*), then A is a fuzzy supra β −nowhere dense set in (X, T^*).

Proof: Let A be a fuzzy supra β -closed set with$-int^*(A) = 0$, in (X, T^*).Then, $\beta - cl^*(A) = A$ and $\beta - int^*(A) = 0$ and hence $\beta - int^*\beta - cl^*(A) = \beta - int^*(A) = 0$ in (X, T^*). Therefore A is a fuzzy supra $\beta-$ nowhere dense set in (X, T^*).

Proposition 3.4: If A is a non-zero fuzzy set in a fuzzy supra topological space (X, T^*), then $A \vee int^*cl^*int^*(A) \leq \beta - cl^*(A)$, in (X, T^*).

Proof: Since $\beta - cl^*(A)$ is a fuzzy β -closed set in $(X, T^*), int^*cl^*int^*\left(\beta - cl^*(A)\right) \leq \beta - cl^*(A)$. Then, $int^*cl^*int^*(A) \leq int^*cl^*int^*\left(\beta - cl^*(A)\right) \leq \beta - cl^*(A)$ and $A \leq \beta - cl^*(A)$ and hence $A \vee int^*cl^*int^*(A) \leq \beta - cl^*(A)$, in (X, T^*).

Proposition 3.5: If A is a fuzzy supra open and fuzzy supra dense set in a fuzzy supra topological space (X, T^*), then A is a fuzzy supra $\beta-$ dense set in (X, T^*).

Proof: Let A be a fuzzy supra open and fuzzy supra dense set in (X, T^*). Then $int^*(A) = A$ and $cl^*(A) = 1$ in (X, T^*). From the proposition 3.4, $A \vee int^*cl^*int^*(A) \leq \beta - cl^*(A)$ and hence $A \vee int^*cl^*(A) \leq \beta - cl^*(A)$. Then, $A \vee int^*(1) \leq \beta - cl^*(A)$. This implies that $A \vee (1) \leq \beta - cl^*(A)$ and $1 \leq \beta - cl^*(A)$. That is, $\beta - cl^*(A)$, in (X, T^*). Therefore A is a fuzzy supra $\beta-$ dense set in (X, T^*).

Proposition 3.6: If A is a fuzzy supra β −nowhere dense set in a fuzzy supra topological space (X, T^*), then $\beta - int^*(A) = 0$, in (X, T^*).

Proof: Let A be a fuzzy supra β −nowhere dense set in (X, T^*). Then, $\beta - int^*\beta - cl^*(A) = 0$, in Now $\beta - int^*(A) \leq \beta - int^*\beta - cl^*(A)$, implies that $\beta - int^*(A) \leq 0$, in (X, T^*). That is, $\beta - int^*(A) = 0$, in (X, T^*).

Proposition 3.7: If A is a non-zero fuzzy set in a fuzzy supra topological space (X, T^*), then $\beta - int^*(A) \leq A \wedge cl^*int^*cl^*(A)$, in (X, T^*).

Proof: Since $\beta - int^*(A)$ is a fuzzy supra β −open set in (X, T^*), $\beta - int^*(A) \leq cl^*int^*cl^*\left(\beta - int^*(A)\right)$. Then $\beta - int^*(A) \leq cl^*int^*cl^*(A)$ (since $\beta - int^*(A) \leq A$) and hence $\beta - int^*(A) \leq A \wedge cl^*int^*cl^*(A)$, in (X, T^*).

Proposition 3.8: If A is a fuzzy supra nowhere dense set in a fuzzy supra topological space (X, T^*), then $\beta - int^*(A) = 0$, in (X, T^*).

Proof: Let A be a fuzzy supra nowhere dense set in (X, T^*). Then, $int^*cl^*(A) = 0$, in (X, T^*).
Then $cl^*int^*cl^*(A) = cl^*(0) = 0$ in (X, T^*). Now by proposition 3.7, $\beta - int^*(A) \leq A \wedge cl^*int^*cl^*(A)$, implies that $\beta - int^*(A) \leq A \wedge 0 = 0$, in (X, T^*). That is, $\beta - int^*(A) = 0$, in (X, T^*).

Proposition 3.9: If A is a fuzzy supra $\beta-$ nowhere dense set in a fuzzy supra topological space (X, T^*) , then $int^*(A) = 0$, in (X, T^*).

Proof: Let A be a fuzzy supra $\beta-$ nowhere dense set in (X, T^*). Then by proposition 3.8, $\beta - int^*(A) = 0$, in (X, T^*). Since $int^*(A) \leq \beta - int^*(A)$, we have $int^*(A) \leq 0$.That is, $int^*(A) = 0$, in (X, T^*).

Proposition 3.9: If A is a fuzzy supra $\beta-$ nowhere dense set in a fuzzy supra topological space (X, T^*) , then $\beta - cl^*(A)$ is a fuzzy supra $\beta-$ nowhere dense set in (X, T^*).

Proof: Let A be a fuzzy supra $\beta-$ nowhere dense set in (X, T^*). Then, $\beta - int^* \beta - cl^*(A) = 0$, in (X, T^*). Now $\beta - int^* \beta - cl^*\left(\beta - cl^*(A)\right) = \beta - int^* \beta - cl^*(A)$ and hence $\beta - int^* \beta - cl^*\left(\beta - cl^*(A)\right) = 0$, in (X, T^*). Therefore $\beta - cl^*(A)$ is a fuzzy supra $\beta-$ nowhere dense set in (X, T^*).

Propostion 3.10: If A is a fuzzy supra $\beta -$ nowhere dense set in a fuzzy supra topological space (X, T^*), then $1 - A$ is a fuzzy supra $\beta -$ dense set in (X, T^*).
Proof: Let If A is a fuzzy supra $\beta -$ nowhere dense set in (X, T^*), then $\beta - int^* \beta - cl^*(A) = 0$, in (X, T^*). Now $1 - \beta - int^* \beta - cl^*(A) = 1 - 0 = 1$ and hence $\beta - cl^* \beta - int^*(A) = 1$ in (X, T^*). But, is a fuzzy supra $\beta -$ dense set in (X, T^*). $\beta - cl^* \beta - int^*(1 - A \leq \beta - cl * 1 - A,$ implies that $1 \leq \beta - cl * 1 - A$. That is, $\beta - cl^*(1 - A) = 1$, in (X, T^*). Therefore then $1 - A$ is a fuzzy supra $\beta -$ dense set in (X, T^*).

Proposition 3.11: If A is a fuzzy supra $\beta -$ nowhere dense set in a fuzzy supra topological space (X, T^*), then $1 - \beta - cl^*(A)$ is a fuzzy supra $\beta -$ dense set in (X, T^*).
Proof: Let A be a fuzzy supra $\beta -$ nowhere dense set in (X, T^*). Then by proposition 3.9, $\beta - cl^*(A)$ is a fuzzy supra $\beta -$ nowhere dense set in (X, T^*). By proposition 3.10, $1 - \beta - cl^*(A)$ is a fuzzy supra $\beta -$ dense set in (X, T^*).

Proposition 3.12: If $A \leq B$ and B is a fuzzy supra $\beta -$ nowhere dense set in a fuzzy supra topological space (X, T^*), then A is also a fuzzy supra $\beta -$ nowhere dense set in (X, T^*).
Proof: Now $A \leq B$, implies that $\beta - cl^*(A) \leq \beta - cl^*(B)$ in (X, T^*). Then $\beta - int^* \beta - cl^*(A) \leq \beta - int^* \beta - cl^*(B)$ in (X, T^*). Since B is a fuzzy supra $\beta -$ nowhere dense set in (X, T^*), $\beta - int^* \beta - cl^*(B) = 0$. Then, $\beta - int^* \beta - cl^*(B) \leq 0$, in (X, T^*). That is, $\beta - int^* \beta - cl^*(B) = 0$, in (X, T^*). Therefore A is a fuzzy supra $\beta -$ nowhere dense set in (X, T^*).

Proposition 3.13: If either $A \ or \ B$ is a fuzzy supra $\beta -$ nowhere dense set in a fuzzy supra topological space (X, T^*), then $A \wedge B$ is a fuzzy supra $\beta -$ nowhere dense set in (X, T^*).
Proof: $\beta - int^* \beta - cl^*(A \wedge B) \leq \beta - int^* [\beta - cl * A \wedge \beta - cl * B = \beta - int * \beta - cl * A \wedge \beta - int * \beta - cl * B, in$ (X, T^*). If either $A \ or \ B$ is a fuzzy supra $\beta -$ nowhere dense set in (X, T^*), then either $\beta - int^* \beta - cl^*(A) = 0$ or $\beta - int^* \beta - cl^*(B) = 0$. Then, either $\beta - int^* \beta - cl^*(A \wedge B) \leq 0 \wedge \beta - int^* \beta - cl^*(B)$ or $\beta - int^* \beta - cl^*(A \wedge B) \leq \beta - int^* \beta - cl^*(A) \wedge 0 = 0$. Therefore $A \wedge B$ is a fuzzy supra $\beta -$ nowhere dense set in (X, T^*).

Proposition 3.14: If A is a fuzzy supra nowhere dense and fuzzy $\beta -$closed set in a fuzzy supra topological space (X, T^*), then A is a fuzzy supra $\beta -$nowhere dense set in (X, T^*).
Proof: Let A be a fuzzy supra nowhere dense and fuzzy $\beta -$closed set in (X, T^*). Since A is a fuzzy supra nowhere dense set in (X, T^*), by proposition 3.7, we have $\beta - int^*(A) = 0$, in (X, T^*). Also since A is a fuzzy supra $\beta -$ closed, $\beta - cl^*(A) = A$, in (X, T^*) and hence $\beta - int^* \beta - cl^*(A) = \beta - int^*(A) = 0$. Therefore A is a fuzzy supra $\beta -$nowhere dense set in (X, T^*).

Proposition 3.15: If A is a fuzzy supra $\beta -$ dense set in a fuzzy supra topological space (X, T^*), then A is a fuzzy supra $\beta -$open set in (X, T^*).
Proof: Let A be a fuzzy supra $\beta -$ dense set in (X, T^*), Then $\beta - cl^*(A) = 1$, in (X, T^*). Since $\beta - cl^*(A) \leq cl^*(A), 1 \leq cl^*(A)$. That is, $cl^*(A) = 1$, in (X, T^*). Now $cl^* int^* cl^*(A) = cl^* int^*(A) = cl^*(1) = 1$ and hence $A \leq cl^* int^* cl^*(A)$, in (X, T^*). Therefore A is a fuzzy supra $\beta -$open set in (X, T^*).

Proposition 3.16: If A is a fuzzy supra nowhere dense set in a fuzzy supra topological space (X, T^*), then A is not a fuzzy supra $\beta -$nowhere dense set in (X, T^*).
Proof: Let A be a fuzzy supra nowhere dense set in (X, T^*). Then, $int^* cl^*(A) = 0$, in (X, T^*). Since $\beta - cl^*(A) \leq cl^*(A)$, $int^*[\ \beta - cl^*(A)] \leq int^*[cl^*(A)]$, in (X, T^*) and hence $int^* \beta - cl^*(A) \leq 0$. That is, $int^* \beta - cl^*(A) = 0$. But $int^* \beta - cl^*(A) \leq \beta - int^* \beta - cl^*(A)$, implies that $0 \leq \beta - int^* \beta - cl^*(A)$ and hence $\beta - int^* \beta - cl^*(A) \neq 0$, in (X, T^*). Therefore A is not a fuzzy supra $\beta -$nowhere dense set in (X, T^*).

Proposition 3.17: If A is a fuzzy supra $\beta -$nowhere dense set in a fuzzy supra topological space (X, T^*), then A is not a fuzzy supra nowhere dense set in (X, T^*).
Proof: Let A be a fuzzy supra $\beta -$nowhere dense set in (X, T^*). Then we have $\beta - int^* \beta - cl^*(A) = 0$, in (X, T^*). Now $int^* \beta - cl^*(A) \leq \beta - int^* \beta - cl^*(A)$, implies that $int^* \beta - cl^*(A) = 0$. But $int^* \beta - cl^*(A) \leq int^* cl^*(A)$ implies that $0 \leq int^* cl^*(A)$ and hence $int^* cl^*(A) \neq 0$, in (X, T^*). Therefore A is not a fuzzy supra nowhere dense set in (X, T^*).

Proposition 3.18: If A is a fuzzy supra nowhere dense and fuzzy supra closed set in a fuzzy supra topological space (X, T^*), then A is a fuzzy supra $\beta -$nowhere dense set in (X, T^*).
Proof: Let A be a fuzzy supra nowhere dense and fuzzy supra closed set in (X, T^*). Then, A is a fuzzy supra nowhere dense and fuzzy supra $\beta -$closed set in (X, T^*) and hence, by proposition3.9, A is a fuzzy supra $\beta -$nowhere dense set in (X, T^*).

Proposition 3.19: If A is a fuzzy supra $\beta -$nowhere dense and fuzzy supra $\beta -$closed set in a fuzzy supra topological space (X, T^*), then A is a fuzzy supra nowhere dense set in (X, T^*).
Proof: Let A be a fuzzy supra $\beta -$nowhere dense and fuzzy supra closed set in (X, T^*). Then, $\beta - int^* \beta - cl^*(A) = 0$ and $cl^*(A) = A$, in (X, T^*). But $\beta - int * A \leq \beta - int * \beta - cl * A$, implies that $\beta - int * A \leq 0.$ That is, $\beta - int^*(A) = 0$, in (X, T^*). we have $int^*(A) \leq \beta - int^*(A)$ and hence $int^*(A) = 0$. Then $int^* cl^*(A) = int^*(A) = 0$ in (X, T^*). Therefore A is a fuzzy supra nowhere dense set in (X, T^*).

Proposition 3.20: If A is a fuzzy supra closed set in a fuzzy supra topological space (X, T^*) then A is a fuzzy supra $\beta -$nowhere dense set if and only if A is a fuzzy supra nowhere dense set in (X, T^*).
Proof: The proof follows from Proposition 3.18 and 3.19.

Definition 3.21: A fuzzy set A in a fuzzy supra topological space (X, T^*) is called a fuzzy supra $\beta -$first category set if $A = \vee_{i=1}^{\infty} (A_i)$, $(A_i)'s$ are fuzzy supra $\beta -$nowhere dense sets in (X, T^*). Any other fuzzy set in (X, T^*) is said to be of fuzzy supra $\beta -$second category.

Definition 3.22: Let A be a fuzzy supra $\beta-$first category set in a fuzzy supra topological space (X,T^*) . Then $1-A$ is called a fuzzy supra $\beta-$residual set in (X,T^*).

4. FUZZY SUPRA β −BAIRE SPACES

Definition 4.1: Let (X,T^*) be a fuzzy supra topological space. Then (X,T^*) is called a fuzzy supra $\beta-$Baire space if $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*).

Example 4.2: Let $X=\{a,b,c\}$. The fuzzy sets $\lambda,\mu,\vartheta,\delta,\alpha,\eta$ are defined on X as follows:

$\lambda: X\to[0,1]$ is defined as

$$\lambda(a)=0.5,\lambda(b)=0.6,\lambda(c)=0.6.$$

$\mu: X\to[0,1]$ is defined as

$$\mu(a)=0.5,\mu(b)=0.5,\mu(c)=0.6.$$

$\vartheta: X\to[0,1]$ is defined as

$$\vartheta(a)=0.4,\vartheta(b)=0.6,\vartheta(c)=0.3.$$

$\delta: X\to[0,1]$ is defined as

$$\delta(a)=0.3,\delta(b)=0.1,\delta(c)=0.7.$$

$\alpha: X\to[0,1]$ is defined as

$$\alpha(a)=0.5,\alpha(b)=0.4,\alpha(c)=0.6.$$

$\eta: X\to[0,1]$ is defined as

$$\eta(a)=0.6,\eta(b)=0.6,\eta(c)=0.7.$$

Then $T^*=\{0,\alpha,\eta,1\}$ is a fuzzy supra topology on X. On computation, one can see that the non-zero fuzzy supra $\beta-$open sets in (X,T^*) are $\lambda,\mu,1-\vartheta,\delta,\alpha,\eta$, 1-$\delta$, 1 and the fuzzy supra $\beta-$nowhere dense sets in (X,T^*) are $1-\lambda,1-\mu,1-\vartheta,\delta$. On computations, $\beta-int^*[1-\lambda\vee 1-\mu\vee 1-\vartheta\vee\delta$]=0. Hence the fuzzy supra topological space (X,T^*) is a fuzzy supra $\beta-$Baire space.

Proposition 4.3: If $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$, where $\beta-int^*(A_i)=0$ and $(A_i)'s$ are fuzzy $\beta-$closed sets in a fuzzy supra topological space (X,T^*), then (X,T^*) is a fuzzy supra $\beta-$Baire space.

Proof: Let $(A_i)'s$ be fuzzy supra $\beta-$closed sets in (X,T^*). Since $\beta-int^*(A_i)=0$, by proposition 3.3, the $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*). Therefore $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*) implies that (X,T^*) is a fuzzy supra $\beta-$Baire space.

Proposition 4.4: If $\beta-cl^*\left(\wedge_{i=1}^{\infty}(A_i)\right)=1$, where$(A_i)'s$ are fuzzy supra $\beta-$dense and fuzzy supra $\beta-$open sets in a fuzzy supra topological space (X,T^*), then (X,T^*) is a fuzzy $\beta-$Baire space.

Proof: Now $\beta-cl^*\left(\wedge_{i=1}^{\infty}(A_i)\right)=1$, implies that $1-\beta-cl^*\left(\wedge_{i=1}^{\infty}(A_i)\right)=0$. Then, $\beta-int^*\big(1-$ $\wedge i=1\infty Ai=0$, in $X,T*$. This implies that $\beta-int$ $*$ $\left(\vee_{i=1}^{\infty}(1-A_i)\right)=0$. Since $(A_i)'s$ are fuzzy supra $\beta-$dense sets in (X,T^*), $\beta-cl^*(A_i)=1$ and $\beta-int^*(1-A_i)=1-\ \beta-cl^*(A_i)=1-1=0$. Hence, $\beta-int^*\left(\vee_{i=1}^{\infty}(1-A_i)\right)=0$,

Where $\beta-int^*(1-A_i)=0$, and $(1-A_i)'s$ are fuzzy supra $\beta-$closed sets in (X,T^*). Then, by proposition 4.3, the fuzzy supra topological space (X,T^*) is a fuzzy supra $\beta-$Baire space.

Proposition 4.5: Let (X,T^*) be a fuzzy supra topological space. Then the following are equivalent:

(1) (X,T^*) is a fuzzy supra $\beta-$Baire space.

(2) $\beta-int^*(A)=0$ for every fuzzy supra $\beta-$first category set A in (X,T^*).

(3) $\beta-cl^*(B)=1$ for every fuzzy supra $\beta-$residual set B in (X,T^*).

Proof: (1) ⇒ (2). Let A be a fuzzy supra $\beta-$first category set in (X,T^*). Then $A=\left(\vee_{i=1}^{\infty}(A_i)\right)$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*). Now $\beta-int^*(A)=\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$ (since (X,T^*) is a fuzzy supra $\beta-$ Baire space). Therefore $\beta-int^*(A)=0$,in (X,T^*).

(2) ⇒ (3) Let B be a fuzzy supra $\beta-$residual set B in (X,T^*). Then, $1-B$ is a fuzzy supra $\beta-$first category set in (X,T^*). By hypothesis, $\beta-int^*(1-B)=0$, in (X,T^*). This implies that $1-\beta-cl^*(B)=0$ and hence $\beta-cl^*(B)=1$, in (X,T^*).

(3) ⇒ (1) Let A be a fuzzy supra $\beta-$first category set in (X,T^*). Then, $A=\left(\vee_{i=1}^{\infty}(A_i)\right)$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*). Since A is a fuzzy supra $\beta-$first category set in (X,T^*), $1-A$ is a fuzzy supra $\beta-$residual set B in (X,T^*). By hypothesis, $\beta-cl^*(1-A)=1$. Then, $1-\beta-int^*(A)=1$, in (X,T^*). This implies that $\beta-int^*(A)=0$, in (X,T^*). Hence $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*), implies that (X,T^*) is a fuzzy supra $\beta-$Baire space.

Definition 4.6: A fuzzy supra topological space (X,T^*) is said a fuzzy supra $\beta-$first category space if $\vee_{i=1}^{\infty}(A_i)=1_x$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*). Otherwise (X,T^*) will be called a fuzzy supra $\beta-$Second category space.

Proposition 4.7: If a fuzzy supra topological space (X,T^*) is a fuzzy supra $\beta-$ Baire space, then (X,T^*) is a fuzzy supra $\beta-$Second category space.

Proof: Let (X,T^*) be a fuzzy supra $\beta-$ Baire space. Then, $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$, where $(A_i)'s$ are fuzzy supra $\beta-$ nowhere dense sets in (X,T^*). Then $\beta-\left(\vee_{i=1}^{\infty}(A_i)\right)\neq 1_x$ [otherwise, $\beta-\left(\vee_{i=1}^{\infty}(A_i)\right)=1_x$, implies that $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=\beta-int^*(1_x)=1_x\neq 0$, a contradiction]. Hence (X,T^*) is a fuzzy supra $\beta-$second category space.

Proposition 4.8: If a fuzzy supra topological space (X,T^*) is a fuzzy supra $\beta-Baire$ space then no non zero fuzzy supra $\beta-$open set in (X,T^*) is a fuzzy supra $\beta-$first category set in (X,T^*).

Proof: Suppose that A is a non-zero fuzzy supra $\beta-$open set in (X,T^*) such that $A=\left(\vee_{i=1}^{\infty}(A_i)\right)$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*). Then, $\beta-int^*(A)=\ \beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)$. Since A is a non-zero fuzzy supra $\beta-$open set in (X,T^*), $\beta-int^*(A)=A$, in (X,T^*). Then $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=A\neq 0$. But this is a contradiction to (X,T^*) being a fuzzy supra $\beta-B$ aire space, in which $\beta-int^*\left(\vee_{i=1}^{\infty}(A_i)\right)=0$, where $(A_i)'s$ are fuzzy supra $\beta-$nowhere dense sets in (X,T^*). Hence it must be that $A\neq\left(\vee_{i=1}^{\infty}(A_i)\right)$. Therefore no non-zero fuzzy supra $\beta-$open set in (X,T^*) is a fuzzy supra $\beta-$first category set in (X,T^*).

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